Homework 1 – Rebah Özkoç - 29207

1- Sketch a composed TM M that performs the computation: (s, ◊ # ω) --|\* M (h, ◊ # ω c)

|  |  |  |  |
| --- | --- | --- | --- |
|  | **Condition** | **TM** | **Comments** |
| M> | -------- | R A |  |
| A | σ = # | L# h |  |
|  | σ ≠ # and σ ≠ $ | R A |  |
|  | σ = $ | R B |  |
| B | σ = $ | x R B | X ∉ Σ |
|  | σ ≠ $ | x L$ σ R C |  |
| C | σ = x | $ R C |  |
|  | σ ≠ x | L# R A |  |

Verbal description:

Head goes to the first symbol of w and enters state A.

State A: If w is e (empty) or it does not contain any $ symbol, it goes back until the first blank symbol which is next to the blip, and it holds.

State A: If head sees a $ symbol while it is going to the left it goes one right and enters to state B.

State B: If the symbol at the right of the symbol $ is also $ head writes x which does not belong to the alphabet and it stays at the state B.

State B: While going to the right if head sees a symbol which is not $ it removes it by putting an x it goes to the left until it sees a $ and writes what is sees and goes one step to the right and enters state C.

State C: In this state the machine deletes the temporary x symbols which does not belong the current alphabet. If head sees an x it writes $ instead of it and goes one step to the right and stays at state C.

State C: If the head sees a symbol which is not x it goes to the left until the first blank symbol which is at the left of the w and it goes one step right and enters state A.

2. Text

Description automatically generated

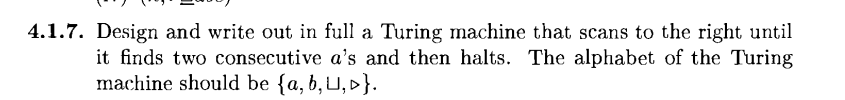
**Solution:**

(q0, ◊ # an a) |-M (q1, ◊ # an-1 a a) |-M (q2, ◊ # an-1 # a) |-M (q0, ◊ # an-2 a # a) |-M (q1, ◊ # an-3 a a # a) |-M

(q2, ◊ # an-3 # a # a) |-M (q0, ◊ # an-4 a # a # a) |-\*M (q0, ◊ # a (#a)n/2 ) |-M (q1, ◊ # a (#a)n/2 )

|-M (h, ◊ # (#a)n/2)

If n is even M halts on the blank but if n is odd M loops forever between the blank next to the blip to right and back.



**Solution:**

This Turing machine semi-decides a language. If there is no two consecutive a’s on the tape it goes right and reads the tape indefinitely.

|  |  |  |
| --- | --- | --- |
|  | **Condition** | **TM** |
| >M | --------- | R A |
| A | σ = a | R B |
|  | σ ≠ a | R A |
| B | σ = a | h |
|  | σ ≠ a | R A |

Diagram

Description automatically generated

**Solution:**

**L U RU a LU a**

(◊ # a a b b #) |-S→

(◊ # a a b b #) |-S→ (◊ # a a b # #) |-S→ (◊ # a a b # #) |-S→ (◊ # a a b # b) |-S→ (◊ # a a b # b) |-S→ (◊ # a a b b b) |-S→

(◊ # a a b b b) |-S→ (◊ # a a # b b) |-S→ (◊ # a a # b b #) |-S→ (◊ # a a # b b b) |-S→ (◊ # a a # b b b) |-S→ (◊ # a a b b b b) |-S→

(◊ # a a b b b b) |-S→ (◊ # a # b b b b) |-S→ (◊ # a # b b b b #) |-S→ (◊ # a # b b b b a) |-S→ (◊ # a # b b b b a) |-S→(◊ # a a b b b b a) |-S→

(◊ # a a b b b b a) |-S→(◊ # # a b b b b a) |-S→(◊ # # a b b b b a #) |-S→ (◊ # # a b b b b a a) |-S→ (◊ # # a b b b b a a) |-S→

(◊ # # a b b b b a a) |-S→ (◊ # # a b b b b a a)

Text

Description automatically generated

**Solution:**

1. It will hold with no directly.

|  |  |  |
| --- | --- | --- |
|  | Condition | TM |
| >M | ----- | hno |

|  |  |  |
| --- | --- | --- |
|  | Condition | TM |
| >M | ---- | R A |
| A | σ = # | hyes |
|  | σ ≠ # | hno |

|  |  |  |
| --- | --- | --- |
|  | Condition | TM |
| >M | ----- | R A |
| A | σ = a | R B |
|  | σ ≠ a | hno |
| B | σ = # | hyes |
|  | σ ≠ # | hno |

|  |  |  |
| --- | --- | --- |
|  | Condition | TM |
| >M | ----- | R A |
| A | σ = # | hyes |
|  | σ = b | hno |
|  | σ = a | R A |

Text, letter

Description automatically generated

1. A two-way infinite tape turing machine is (Q, Σ, t, s, H) where t is the transition function.

A configuration (q, u a v) belongs to Q x ((Σ – {#}) Σ\* U {#}) x (Σ\* (Σ – {#}) U {e})

M semidices L means if the input w belongs to L then M will hold. If the w does not belong to L it may continue to work indefinitely.

w belongs to L : (s, #, w) |-\*M (h, x, y) for some h belongs to H

1. M a k-tape Turing machine computes f if Σ\*0 -> Σ\*0 for all w belongs to Σ\*0

(q0, ◊ # w, ◊ # , … , ◊ # ) |-M\* (h, ◊ # f(w), w2 , … wk) where h belongs to H.

Text

Description automatically generated

1. A k-head Turing machine with a single one-way infinite tape is defined as M = (Q, Σ, t, s, H), where Q, Σ, s and H are same with the standard Turing machine and t is the transition function goes from (Q-H) x Σk to Q x (Σ U {<- , ->}k .
2. The configuration of the machine is an element of Q x ◊Σ\* x Nk

(q, uk ak vk) where q is the current state of the machine which belongs to Q.

uk is the string that is at the left of the kth head ( this can be the blip symbol.)

vk is the string that is at the right of the kth head. (this can be blank or empty.)

ak is the symbol under the hth head.

Text, letter

Description automatically generated

A Turing machine with a 2 dimensional infinite tape is defined as M = (Q, Σ, t, s, H), where Q, Σ, s and H are same with the standard Turing machine and t is the transition function goes from (Q-H) x Σ to Q x (Σ U {<- , ->, up, down} .